

Name: \_\_\_\_\_ Code: \_\_\_\_\_

**Nova School of Business and Economics**  
**Macroeconomics, 1103 - 1st Semester 2013-2014**  
**Prof. André C. Silva**  
**TAs: João Vaz, Paulo Fagandini, and Pedro Freitas**

### Midterm 1

Maximum points: 20. Time: 1h. Pages: 10. The exam is closed books, closed notes. No calculators are allowed. You may write on the back of the pages if you need space.

1. (2 pts) The growth rate of hours of work in a country during a certain period was 1 percent per year and the growth rate of output was 1.2 percent per year. What is your estimate for the growth rate of capital during the same period?

Hint: Use the production function  $Y = K^a N^{1-a}$ , where  $Y$  denotes output,  $K$  capital,  $N$  hours of work, and  $a$  is the share of capital on total income (in your estimation, you should assign a reasonable value for  $a$ ).

2. (2 pts) Select the best answer.

a. In the last two years, the rate of inflation in Portugal has been (negative and close to zero / around zero / small but positive).

b. Suppose that someone has high income today and small income in the future. If the interest rate today increases, then the welfare of this person will probably (decrease / increase).

c. Investment in Portugal as a fraction of GDP (has been decreasing since 2000 / increased from 2000 to 2008, but decreased after the 2008 crisis / decreased after the 2008 crisis but has been recovering ever since).

d. It is possible to reconcile the evidence on the long-run behavior of hours of work plus school hours if (the substitution effect of wages is larger than the income effect / the substitution and income effects cancel each other / the income effect of wages is larger than the substitution effect).

3. (2 pts) Consider the demand for labor,  $N^d$ . What should happen with  $N^d$  if the current technology improves? Justify briefly. Use graphs  $w \times N$ .

4. (6 pts) Obtain the effects of a large increase in government expenditures. Simplify by using a one-period model and by assuming that taxes are lump sum.
- a. (4 pts) Obtain the effects on consumption and labor. Use the diagram  $c \times l$ , where  $l$  denotes leisure. Justify.

b. (2 pts) According to your results, how does GDP change with the increase in government expenditures? Is there a multiplier effect? Explain.

5. (8 pts) A credit card company hired you to analyze the behavior of savings of a group of clients. You start by stating that the clients, that is, consumers in your model, have preferences

$$\log c_1 + \beta \log c_2, \quad (1)$$

where  $c_1$  and  $c_2$  refer to consumption in periods 1 and 2, and  $0 < \beta < 1$ .  $\log$  is the logarithm in the base  $e$ . Given income in periods 1 and 2,  $y_1$  and  $y_2$ , savings  $s$ , and the real interest rate  $r$ , the budget constraints are

$$c_1 + s = y_1, \quad (2)$$

for the first period, and

$$c_2 = y_2 + (1 + r) s, \quad (3)$$

for the second period. The two budget constraints imply the single budget constraint

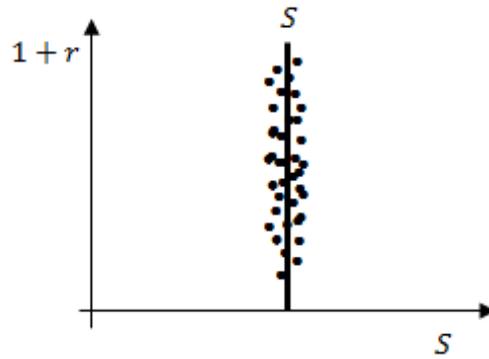
$$c_1 + \frac{1}{1+r} c_2 = y_1 + \frac{1}{1+r} y_2. \quad (4)$$

a. (2 pts) Write the maximization problem of the consumers and its first order conditions.

b. (2 pts) Obtain the optimal values of  $c_1$  and  $s$ .

c. (2 pts) Represent the solution in the diagram  $c_2 \times c_1$  ( $c_1$  in the horizontal axis), consider  $y_1 > y_2$ . Using the diagram, show the effects of an increase in  $r$ .

d. (2 pts) The credit card company has a database on savings of a group of clients for different interest rates. You plot the data and obtain the result in the figure below. Given the evidence in the figure, which value of  $y_2$  should you choose for your analysis? Justify.



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## SOLUTION SKETCH

### Question 1

1. (2 pts) The growth rate of hours of work in a country during a certain period was 1 percent per year and the growth rate of output was 1.2 percent per year. What is your estimate for the growth rate of capital during the same period? Hint: Use the production function  $Y = K^a N^{1-a}$ , where  $Y$  denotes output,  $K$  capital,  $N$  hours of work, and  $a$  is the share of capital on total income.

#### Answer

Taking logs on the production function  $Y = K^a N^{1-a}$ , we obtain

$$\log Y = a \log K + (1 - a) N.$$

Then, taking derivatives with respect to time implies

$$\frac{\dot{Y}}{Y} = a \frac{\dot{K}}{K} + (1 - a) \frac{\dot{N}}{N},$$

where a dot over the variables denotes the derivative with respect to time. For example,  $\frac{\dot{Y}}{Y} = \frac{dY/dt}{Y}$ . Each ratio  $\frac{\dot{x}}{x}$  expresses the growth rate of  $x$  during the period.

Substituting the values of the question yields

$$1.2 = a \frac{\dot{K}}{K} + (1 - a) 1.$$

Assign a reasonable value for  $a$ . For example,  $a = \frac{1}{3}$ . This implies

$$1.2 = \frac{1}{3} \frac{\dot{K}}{K} + \frac{2}{3} \Rightarrow \frac{\dot{K}}{K} = 3 \left( \frac{1.2}{3} - \frac{2}{3} \right) \Rightarrow \frac{\dot{K}}{K} = 1.6.$$

Therefore, an estimate for the growth rate of capital during the period is 1.6% per year.

## Question 2

2. (2 pts) Select the best answer.

a. In the last two years, the rate of inflation in Portugal has been (negative and close to zero / around zero / **small but positive**).

b. Suppose that someone has high income today and small income in the future. If the interest rate today increases, then the welfare of this person will probably (decrease / **increase**). [Note: If income is high today and small in the future, this person probably has positive savings. As a result, an increase in interest rates will likely increase welfare for this person.]

c. Investment in Portugal as a fraction of GDP (**has been decreasing since 2000** / increased from 2000 to 2008, but decreased after the 2008 crisis / decreased after the 2008 crisis but has been recovering ever since).

d. It is possible to reconcile the evidence on the long-run behavior of hours of work plus school hours if (the substitution effect of wages is larger than the income effect / **the substitution and income effects cancel each other** / the income effect of wages is larger than the substitution effect).

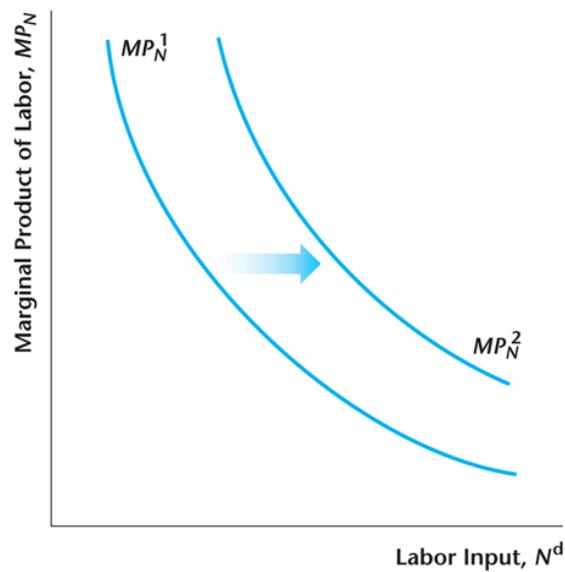
### Question 3

3. (2 pts) Consider the demand for labor,  $N^d$ . What should happen with the demand for labor if the current technology improves? Justify. Use graphs  $w \times N$ .

#### Answer

The demand for labor is such that the marginal productivity at  $N^d(w)$  is equal to the wage rate.  $N^d$  is a decreasing function of  $w$  as the marginal productivity decreases when  $N^d$  increases.

An improvement in the technology (an increase in  $z$  with the notation used in class) makes firms more productive. So, for each wages, firms will be willing to hire more people. This effect makes the demand for labor shift rightward. Figure 4.17 p. 144 of the book shows the effect of an improvement in the technology. The figure is reproduced below.



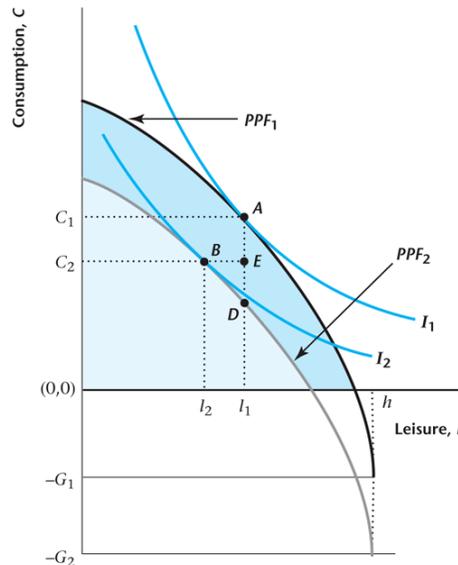
### Question 4

4. (6 pts) Obtain the effects of a large increase in government expenditures. Simplify by using a one-period model and by assuming that taxes are lump sum.

a. (4 pts) Obtain the effects on consumption and labor. Use the diagram  $c \times l$ , where  $l$  denotes leisure. Justify.

#### Answer

An increase in government expenditures makes the production possibilities frontier shift downward in a parallel way in the diagram  $c \times l$ . This change makes consumption and leisure decrease. As leisure decreases, labor increases, as  $N = h - l$ . See pages 169-171 of the book for a detailed explanation. The relevant graph is in Figure 5.6 of the book, reproduced below.



b. (2 pts) According to your results, how does GDP change with the increase in government expenditures? Is there a multiplier effect? Explain.

#### Answer

We have seen from the analysis above that labor increases. As production is given by  $Y = zF(K, N)$ , and increase in  $N$  will make production  $Y$  increase. Note that  $z$  and  $K$  are constant. Therefore, GDP increases with the increase in government expenditures.

To study whether there will be a multiplier effect, consider the definition of the multiplier, given by  $\frac{\Delta Y}{\Delta G}$ . We have seen that  $\Delta Y > 0$ . Then,  $\frac{\Delta Y}{\Delta G} > 0$  and the

multiplier is at least positive. There is a multiplier effect if  $\frac{\Delta Y}{\Delta G} > 1$ . In this case, an increase in one unit of government expenditures increases output by more than one unit. We have that  $Y = C + G$  in the case considered. Therefore,  $\Delta Y = \Delta C + \Delta G$ , which implies  $\frac{\Delta Y}{\Delta G} = \frac{\Delta C}{\Delta G} + 1$ . As consumption decreases,  $\frac{\Delta C}{\Delta G}$  is negative. As a result,  $\frac{\Delta Y}{\Delta G} < 1$ . As the multiplier is smaller than one, there is not a multiplier effect.

### Question 5

5. (8 pts) A credit card company hired you to analyze the behavior of savings of a group of clients. You start by stating that the clients, that is, consumers in your model, have preferences

$$\log c_1 + \beta \log c_2, \quad (1)$$

where  $c_1$  and  $c_2$  refer to consumption in periods 1 and 2, and  $0 < \beta < 1$ .  $\log$  is the logarithm in the base  $e$ . Given income in periods 1 and 2,  $y_1$  and  $y_2$ , savings  $s$ , and the real interest rate  $r$ , the budget constraints are

$$c_1 + s = y_1, \quad (2)$$

for the first period, and

$$c_2 = y_2 + (1 + r)s, \quad (3)$$

for the second period. The two budget constraints imply the single budget constraint

$$c_1 + \frac{1}{1+r}c_2 = y_1 + \frac{1}{1+r}y_2. \quad (4)$$

a. (2 pts) Write the maximization problem of the consumers and its first order conditions.

#### Answer

Using the budget constraint in present value (or the intertemporal budget constraint), the maximization problem is

$$\max_{c_1, c_2} \log c_1 + \beta \log c_2$$

subject to

$$c_1 + \frac{1}{1+r}c_2 = y_1 + \frac{1}{1+r}y_2.$$

Construct the Lagrangian  $L = \log c_1 + \beta \log c_2 + \lambda \left( y_1 + \frac{1}{1+r}y_2 - c_1 - \frac{1}{1+r}c_2 \right)$ .

The first order conditions are, then,

$$\frac{1}{c_1} = \lambda$$

$$\beta \frac{1}{c_2} = \lambda \frac{1}{1+r}$$

and the constraint itself,  $c_1 + \frac{1}{1+r}c_2 = y_1 + \frac{1}{1+r}y_2$ .

b. (2 pts) Obtain the optimal values of  $c_1$  and  $s$ .

The FOCs imply that

$$\frac{c_2}{c_1} = \beta(1+r).$$

Substituting  $c_2 = \beta(1+r)c_1$  in the budget constraint implies

$$c_1 + \frac{1}{1+r}\beta(1+r)c_1 = y_1 + \frac{1}{1+r}y_2$$

$$\Rightarrow c_1 = \frac{1}{1+\beta} \left( y_1 + \frac{1}{1+r}y_2 \right).$$

As  $s = y_1 - c_1$ , we have that

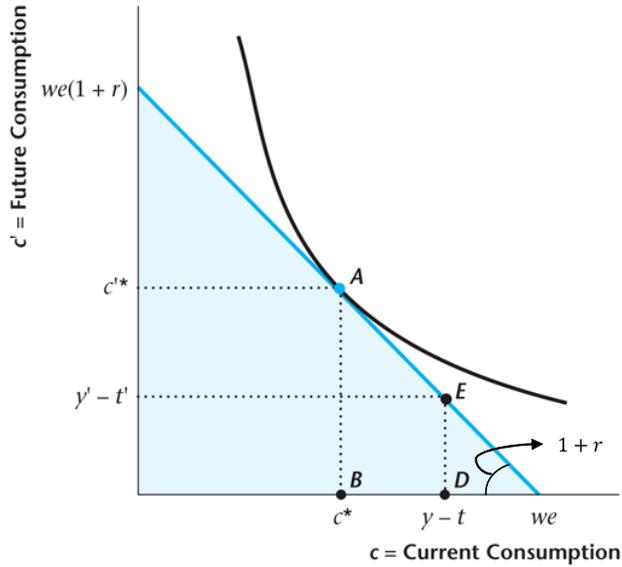
$$s = \frac{1}{1+\beta} \left( \beta y_1 - \frac{1}{1+r}y_2 \right).$$

For the record,  $c_2 = \beta(1+r)c_1 \Rightarrow c_2 = \frac{\beta(1+r)}{1+\beta} \left( y_1 + \frac{1}{1+r}y_2 \right)$ .

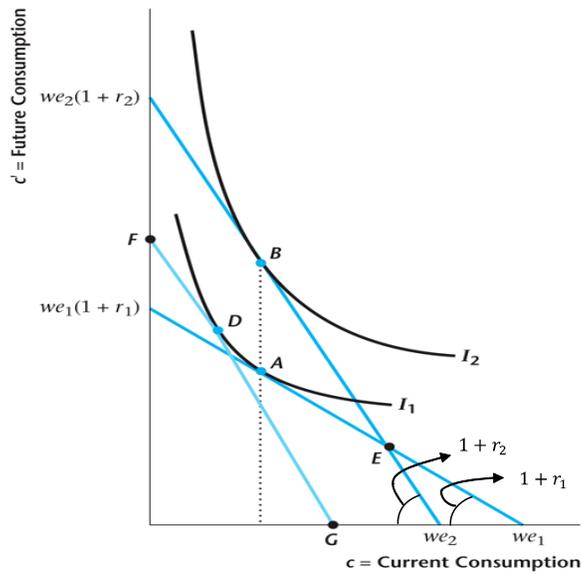
c. (2 pts) Represent the solution in the diagram  $c_2 \times c_1$  ( $c_1$  in the horizontal axis), consider  $y_1 > y_2$ . Using the diagram, show the effects of an increase in  $r$ .

**Answer**

The solution is represented in the diagram below, which reproduced figure 9.3 of the book, p. 318, with the information on the interest rate added to the figure. For this case,  $t$  and  $t'$  are equal to zero. It is important to reflect the case that  $y_1 > y_2$ . It is also important to indicate where the consumer will consume. It is likely that the consumer will be a lender, as shown in the figure.



An increase in  $r$  makes the budget constraint rotate clockwise around the endowment point  $E$ . Consumption  $c_2$  will increase. However, consumption  $c_1$  may increase or decrease, depending on the substitution and income effects. As a result, savings may increase or decrease. Figure 9.13 of the book on p. 332, reproduced below, summarizes the effects. I added the indications of the interest rates. In the graph,  $r_2 > r_1$ .



d. (2 pts) The credit card company has a database on savings of a group of clients for different interest rates. You plot the data and obtain the result in the figure below. Given the evidence in the figure, which value of  $y_2$  should you choose for your analysis? Justify.

**Answer**

The figure on the exam shows that an increase in  $r$  does not affect savings. Our result for savings obtained in  $b$  is  $s = \frac{1}{1+\beta} \left( \beta y_1 - \frac{1}{1+r} y_2 \right)$ . If  $y_2 > 0$ , an increase in interest rate implies an increase in savings (the substitution effect prevails). Mathematically,  $\frac{\partial s}{\partial r} = \frac{1}{1+\beta} \frac{1}{(1+r)^2} y_2$ , which is positive if  $y_2 > 0$ .

If  $y_2 = 0$ , then  $s = \frac{\beta}{1+\beta} y_1$ . In this case, savings are not affected by a change in the interest rate, following the behavior shown in the figure. Therefore, given the evidence in the figure, the value that should be chosen for  $y_2$  is  $y_2 = 0$ .