

**Time value of money**

You asked for a bank loan of 15,000€ and the bank will charge you a 6% stated annual interest rate for 5 years of monthly installments, starting next month. What is the value of the installment?

$$1) \frac{I}{\frac{6\%}{12}} \times \left[ 1 - \frac{1}{\left(1 + \frac{6\%}{12}\right)^{5 \times 12}} \right] = 15,000 \Leftrightarrow I = 289.89$$

289.89€

What is the value of reimbursement in the 1<sup>st</sup> installment of the loan presented in question 1?

$$2) \text{Interest}_1 = \frac{6\%}{12} \times 15,000 = 75$$

$$\text{Reimbursement}_1 = 289.89 - 75 = 214.89$$

214.89€

For the loan in question 1, what is the value of interest paid during the last year of installments (5th year)?

$$3) D_{4Y} = \frac{289.89}{\frac{6\%}{12}} \times \left[ 1 - \frac{1}{\left(1 + \frac{6\%}{12}\right)^{12}} \right] = 3,369.40,$$

$$\text{Reimbursement in Year 5} = D_{4Y} - D_{5Y} = 3,369.40 - 0 = 3,369.40$$

$$\text{Total installments paid during year 5} = 12 \times 289.89 = 3,479.90$$

$$\text{Interest paid in Year 5} = 3,479.90 - 3,369.40 = 110.50$$

110.50€

If you start to pay back the same loan at the beginning of next year (instead of starting one month from now), with the same number of installments, what is the new value of the monthly installment? Present your answer in euros with 4 decimal places (e.g. 1.2345€).

$$4) \frac{\frac{I}{\frac{6\%}{12}} \times \left[ 1 - \frac{1}{\left(1 + \frac{6\%}{12}\right)^{5 \times 12}} \right]}{\left(1 + \frac{6\%}{12}\right)^{11}} = 15,000 \Leftrightarrow I = 306.35$$

306.35€

Mr. Muttley expects to retire in 50 years and he thinks he will need 1,000€ in each month during retirement, for 20 years (he will start spending his retirement at the end of the first month of retirement). If he wants to make annual deposits, with an effective annual rate of 7%, how much should he deposit yearly until he retires, starting next year?

$$5) r_{\text{monthly}}^{\text{eq}} = (1+7\%)^{\frac{1}{12}} - 1 = 0.5654\%$$

$$\text{PV of retirement} = \frac{1,000}{0.5654\%} \times \left[ 1 - \frac{1}{(1+0.5654\%)^{20 \times 12}} \right] = 4,452.49$$

$$\text{PV of deposits} = 4,452.49 = \frac{D}{7\%} \times \left[ 1 - \frac{1}{(1+7\%)^{50}} \right] \rightarrow D = 322.63$$

322.63€

**Bonds**

Use the following info for questions 6-9: The infamous Microsoles, just issued today three zero-coupon bonds with the following yields:

Maturity	Yields
1 Year	3%
2 Year	?%
3 Year	6%

What is the yield of the 2-year zero coupon bond if it is currently trading at 90.70%?

$$6) B = \frac{100}{(1+S_2)^2} = 90.70 \rightarrow S_2 = 5.0017\%$$

5.0017%

What is the forward rate for a two year bond one year from today ( $f_{1,3}$ )?

$$7) f_{1Y,3Y} = \left[ \frac{(1+6\%)^3}{(1+3\%)^1} \right]^{\frac{1}{2}} - 1 = 7.5326\%$$

7.5326%

Microsoles also issued a 2-year annual coupon bond with a coupon rate of 10%. How much is this bond trading for? Coupon is annual.

$$8) B = \frac{10\%}{(1+3\%)^1} + \frac{100\%+10\%}{(1+5.0017\%)^2} = 109.4787\%$$

109.4787%

If the yield to maturity on the 2-year annual coupon bond from question 8 was 8%, what would be the clean price of this bond three months after its issuance?

$$9) CP = DP - AI = \frac{10\%}{(1+8\%)^{0.75}} + \frac{100\%+10\%}{(1+8\%)^{1.75}} - 10\% \times \frac{3}{12} = 105.5785\% - 2.5\% = 103.0785\%$$

103.0785%

### Stocks

Mango Inc. is about to pay a 1 euro dividend. Investors expect this firm to grow at a 5% rate next year and 2% thereafter. What is the price of this stock if the discount rate is 21%?

$$10) S = 1 + \frac{1 \times (1 + 5\%)}{(1 + 21\%)} + \frac{\frac{1 \times (1 + 5\%) \times (1 + 2\%)}{21\% - 2\%}}{(1 + 21\%)} = 6.52$$

6.52€

What is the price-earnings ratio of a stock with zero growth and a payout ratio of 100% in the next two years and a growth rate of 2% and payout ratio of 30% thereafter? Assume that the discount rate is 10%.

$$11) PER = \frac{P}{EPS_1} = \frac{\frac{EPS_1 \times 100\%}{(1+10\%)} + \frac{EPS_1 \times 100\% + \frac{EPS_1 \times 30\%}{10\% - 2\%}}{(1+10\%)^2}}{EPS_1} = 4.8347$$

4.8347€

An Oil company has oil rigs that generate 1€ per share in earnings every year. Because they are currently drilling only on the existing rigs, they have no investment needs and for this reason are paying out all their earnings to stock holders. What would be the stock price, if they have just announced that they found a new oil rig that will have a 25% return on equity and will start exploring it in 5 years? For that, they will retain all earnings 5 years from today. Discount rate is 10%.

$$12) S = \frac{EPS}{r} + NPVGO = \frac{1}{10\%} + \frac{\left[ -1 + \frac{25\% \times 1}{10\%} \right]}{(1+10\%)^5} = 10.9314$$

10.9314€

Specific Motors (SM) just announced total earnings of €50,000. The firm is expected to grow 5% forever; it retains 60% of its earnings, it has 100,000 shares outstanding. What is SM's net present value of growth opportunities (NPVGO) if discount rate is 10%?

$$13) S = \frac{EPS}{r} + NPVGO \Leftrightarrow \frac{0.5 \times (1 + 5\%) \times (1 - 60\%)}{10\% - 5\%} = \frac{0.5}{10\%} + NPVGO \Leftrightarrow NPVGO = -0.8$$

-0.80€

### Capital budgeting

Use the following info for questions 14-17: Guilherme Stephens wants to open a new factory to produce glass bottles. This factory will only operate during three years. He will need to invest in machinery that requires an immediate investment of 90,000€, which should be fully depreciated in three years. These machines will require 10,000€ of maintenance costs in each year. The appropriate discount rate is 10%.

What is the pre-tax Equivalent Annual Cost of this machinery?

$$14) PV = 90,000 + \frac{10,000}{10\%} \times \left[ 1 - \frac{1}{(1+10\%)^3} \right] = 114,868.52 = \frac{EAC}{10\%} \times \left[ 1 - \frac{1}{(1+10\%)^3} \right] \Leftrightarrow EAC = 46,190.33$$

46,190.33€

With these machines, Stephens is expecting to produce 1,000,000 bottles in each year, and he will sell them for 0.20€ each. Besides the maintenance costs, the company needs to spend additional 70,000€ in other operating costs every year. Some products will be sold with credit to clients, and this company will have, on average, 3 months in account receivables at the end of the year, in the first two years. There will be no inventories or account payables during the project. By the end of the third year, there will be no net working capital left. The marginal tax rate is 30% and the discount rate is 10%.

(values in €)	0	1	2	3
Revenues		200,000	200,000	200,000
Maintenance costs		10,000	10,000	10,000
Other operating costs		70,000	70,000	70,000
Depreciation		30,000	30,000	30,000
<b>EBIT</b>		<b>90,000</b>	<b>90,000</b>	<b>90,000</b>
Taxes		27,000	27,000	27,000
<b>NOPAT</b>		<b>63,000</b>	<b>63,000</b>	<b>63,000</b>
<b>OCF</b>		<b>93,000</b>	<b>93,000</b>	<b>93,000</b>
CAPEX	-90,000			
(NWC)	0	$\frac{3}{12} \times 200,000 = 50,000$	50,000	0
Inv. In NWC	0	-50,000	0	50,000
<b>Inv. CF</b>	<b>-90,000</b>	<b>-50,000</b>	<b>0</b>	<b>50,000</b>
<b>Free CF</b>	<b>-90,000</b>	<b>43,000</b>	<b>93,000</b>	<b>143,000</b>

15) What is the free cash flow of year 1? 43,000€

16) What is the NPV of the project?  

$$NPV = -90,000 + \frac{43,000}{(1+10\%)^1} + \frac{93,000}{(1+10\%)^2} + \frac{143,000}{(1+10\%)^3} = 133,388.43$$
133,388.43€

17) What is the profitability index of the project?  

$$PI = \frac{NPV}{I_0} + 1 = \frac{133,388.43}{90,000} + 1 = 2.48$$
2.48

### Risk and Return

Use the following info for questions 18-19: Hyunnight stocks presented the following returns in the past 4 years.

Year	Return
1	-6%
2	-3%
3	-15%
4	10%

18) What is the holding period return of an investor who purchased Hyunnight's stock at the beginning of year 1 and sold it at the end of year 3?  

$$\text{Return} = (1-6\%) \times (1-3\%) \times (1-15\%) - 1 = -22.4970\%$$
-22.4970%

19) What was the annualized rate of return for an investor who bought the stock at the beginning of year 1 and sold it by the end of year 4?  

$$\text{Annualized Return} = [(1-6\%) \times (1-3\%) \times (1-15\%) \times (1+10\%)]^{\frac{1}{4}} - 1 = -3.9101\%$$
-3.9101%

20) Stock XYZ has an expected return of 10% and standard deviation of 20% and stock ABC has an expected return of 15% and standard deviation of 30%. The correlation coefficient between both stock returns is 0.3. If you have a portfolio with 6,000€ invested in stock XYZ and 9,000€ invested in stock ABC, what is the standard deviation of your portfolio? (Hint:  

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 \sigma_A \sigma_B w_A w_B \rho_{A,B}$$

$$w_{XYZ} = \frac{6,000}{6,000+9,000} = 40\%; w_{ABC} = \frac{9,000}{6,000+9,000} = 60\%$$

$$\sigma_p = \sqrt{0.4^2 \times 0.2^2 + 0.6^2 \times 0.3^2 + 2 \times 0.3 \times 0.2 \times 0.3 \times 0.4 \times 0.6} = 21.7807\%$$
21.7807%